STATISTICS - MODULE 12122

EXERCISE 4 - STATISTICAL INFERENCE AND PROPERTIES OF ESTIMATORS

The starred question will be discussed in tutorials in week 9, so it is important that you attempt this before your tutorials. This is an important exercise so should you not be able to do all the questions in term - time, please attempt them in the Easter vacation. Solutions to those not covered in tutorials will be provided in week 1 of the Summer term.

1. A marketing research department conducted a random sample test market for a new cheese-flavoured snack food. A random sample of 398 people included 143 who said that they would definitely buy the product. Determine (i) a point estimate and (ii) a 90% confidence interval for the population proportion who would purchase the product.

2. Records from a large dental practice showed that during 1997 the number of minutes per visits spent in the dentist's chair could be taken to be Normally distributed with mean 14.5 minutes and standard deviation 2.9 minutes. In 1998 a random sample of 16 consultations gave the following times in minutes.

   13.2, 18.7, 14.9, 12.1, 11.6, 17.2, 10.6, 9.4, 14.6, 12.9,
   11.2, 13.5, 12.9, 11.8, 14.1, 12.5

   Summary statistics are $\sum X = 211.2$ and $\sum X^2 = 2871.44$ where X is the time in minutes.

   (a) Obtain the mean and standard deviation of the sample and hence give an unbiased point estimate of m.
   (b) Obtain a 95% confidence interval for m assuming the population standard deviation for 1998 is still 2.9 minutes as in 1997.
   (c) Obtain a 99% one-sided interval for m which indicates the maximum likely value of m assuming the population standard deviation for 1998 is unknown.
   (d) It is claimed by the dental practice that the average (mean) number of minutes spent in the dentist's chair has decreased between 1997 and 1998. Using your result from (c) test this claim at a 1 % level of significance.

3. In U.S.A. most studies indicate that for long periods of time mean rates of return on common stocks have exceeded mean rates on bonds and other fixed dollar assets. Since stocks are more risky than other fixed dollar assets, we are thus offered clear evidence of the assumed relationship between return and market risk. The data in the accompanying table provide the mean and standard deviation of annual returns for randomly chosen securities within each of four asset classes for the period 1981-1990.

Asset Class	Number of Securities	Mean Annual Return	Standard Deviation of Annual Return (Risk)
Common Stocks	50	10.57	19.05
Corporate bonds	35	4.38	5.25
Government securities	30	3.47	3.87
Municipal bonds	35	2.51	8.76

   (a) Assume that the rates of return are Normally distributed for each of the asset classes. Find a 95% confidence interval for the mean annual return for each asset class for the period 1981-1990.

   (b) What do your results in (a) suggest about the asset class which offers :
      (i) the greatest opportunity of a negative return,
      (ii) the greatest opportunity for the highest gain?

   (c) Let m_{CB} be the mean rates of return on corporate bonds

Test the hypothesis $H_0 : m_{CB} = 8$ against $H_1 : m_{CB} \neq 8$ taking $\alpha = 0.05$.

4. A simple random sample of size two, X_1, X_2 is drawn from a population with mean m and variance s^2. Consider the following estimators of m.

$$\bar{m}_1 = 0.7X_1 + 0.3X_2 \qquad \bar{m}_2 = 0.5X_1 + 0.5X_2$$
$$\bar{m}_3 = 0.7X_1 + 0.4X_2 \qquad \bar{m}_4 = 0.3X_1 + 0.6X_2$$

(a) Which of these estimators is an unbiased estimator of m?

(b) Find the variance of each estimator in terms of s^2 and comment on which one has the smallest variance.

(c) Among the unbiased estimators, which has the smallest variance?

5*. Suppose T_1 and T_2 are independent unbiased estimators of a parameter γ and $Var(T_1) = s_1^2$, $Var(T_2) = s_2^2$

(a) Show that $\alpha T_1 + (1-\alpha) T_2$ is also an unbiased estimator of γ.

(b) Find the value of α that minimises the variance of $\alpha T_1 + (1-\alpha) T_2$.

(This is called the best linear unbiased estimator (B.L.U.E) of γ, which is very important in Econometrics).

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