UNIVERSITY OF BRISTOL : DEPARTMENT OF ECONOMICS

STATISTICS - MODULE 12122

EXERCISE 2 - SPECIAL DISTRIBUTIONS

You should attempt as many relevant questions as possible **before** your next tutorial. Although you should attempt all questions, however easy or difficult they appear, only starred questions will be discussed in tutorials, so it is important that you especially attempt these before your tutorials. You are asked to do **Questions 2, 6** and **8** and hand them in by **5 p.m. Friday, 25th February.** Solutions to those not covered in tutorials will be provided later on in the term.

1. *(i) Consider a consumer's poll where consumers are asked if they prefer a particular brand of coffee called *Goldtaste*. Let a random variable X be defined as follows:

$X = 1$ if consumer prefers *Goldtaste*.
$X = 0$ if consumer prefers another brand.

Suppose further that the probability that a consumer prefers *Goldtaste* is p ($0 < p < 1$) so that probability that consumer prefers another brand is $(1 - p)$. Then X is known as a <u>Bernoulli</u> random variable with parameter p and the probability function of X can be written as

$$p(x) = \text{P}(X = x) = p^x(1-p)^{1-x} \qquad x = 0 \text{ or } 1.$$

(a) Show that $\text{E}(X) = p$ and $\text{Var}(X) = p(1 - p)$.

(b) Suppose n consumers independently take part in this poll and Y is the number of consumers who prefer *Goldtaste*, so that $Y = X_1 + X_2 + X_3 + \dots\dots\dots + X_n$ where X_i ($i = 1,2,\dots.n$) are Bernoulli random variables with parameter p. Show that $\text{E}(Y) = np$ and $\text{Var}(Y) = np(1 - p)$. What distribution has Y got?

(ii) Suppose $X \sim \text{U}[a , b]$, then show that

$$E(X) = \frac{(b+a)}{2} \qquad \text{and} \qquad \text{Var}(X) = \frac{(b-a)^2}{12}.$$

2. Experience shows that 40% of people who speculate on the stock market lose money. If 15 people in a large company independently play the stock market, what is the probability that ;
(a) none of them lose their money? (b) at most 3 people lose their money?
(c) between 5 and 10 people inclusive lose their money?

How many people on average will lose money?

*3. In the qualifying round of a diving competition each diver has to execute 5 prescribed dives and has to perform satisfactorily in at least 2 to qualify. For a diver who has probability 0.75 of performing successfully in any one of his 5 independent dives what are the probabilities (a) that he will qualify? (b) that he will require his final dive to qualify?

Suppose the English team consists of 4 divers each having independently the same performance ability as the diver described above. What is the probabilty that at least 3 of the team will qualify?

*4. A common model for the distribution of incomes is one based on the Pareto distribution which has a distribution function given by $F(x)$ where

$$F(x) = 1 - \left[\frac{A}{x}\right]^{\alpha} \quad \text{if } x \geq A \qquad \text{and } F(x) = 0 \text{ if } x < A \quad \text{where } A, \alpha > 0$$

(a) Sketch $F(x)$.
(b) Derive the density function of X and sketch it. (Note that $F(x)$ is zero for $x < A$).
(c) Show that $E\left(X^r\right) = \dfrac{a}{(a-r)}.A^r$ provided $r < a$, and hence find an expression for the mean and variance of the Pareto distribution.
(d) Given data on a particular distribution of incomes, how could you tell whether the Pareto distribution provided an adequate model for your income data?

5. If Z ” N(0,1) , use tables to find (a) the following probabilities:

(i) P(Z < 1.915) (ii) P(Z < 2.53) (iii) P(Z > -1.324) (iv) P(Z > -1.12)
(v) P(1.316 < Z < 2.058) (vi) P(-1.251 < Z < 1.371)

(b) to find z such that

(i) P(Z > z) = 0.05 (ii) P(-z < Z < z) = 0.50 (iii) P(Z ⇔ -z or Z > z) = 0.01
(iv) P(-z ⇔ Z ⇔ z) = 0.90 (v) P(Z ⇔ z) = 0.01.

6. A machine produces jars of coffee and the weights of the jars are normally distributed with mean 260 grams and standard deviation of 4 grams.
(i) What percentage of jars will fall below the weight declared on the label of 250 grams?
(ii) What is the probability that if two jars are selected at random, at least one of them will be below the declared weight?

7. Monthly sales in *Timberland* hardware store are normally distributed with a mean of £8000 and a standard deviation of £2000.
(i) Find the lower limit such that sales will be lower than this 10% of the time.
(ii) Find the upper limit such that sales will exceed this boundary 8% of the time.
(iii) Determine a symmetric interval about the mean that will include 92% of the sales data.

8. Packets of semolina are nominally 226g. in weight. The actual weights have a Normal distribution with μ = 230g and σ = 1.50g. What is the probability that a packet is underweight?

A decision is taken that the probability of an underweight packet should not exceed 0.001. To change the distribution of weights of the semolina packets to conform to this decision, two methods are considered:

(a) to increase μ, leaving σ unaltered;
(b) to improve the packing machine, thus reducing σ, while leaving μ unaltered.

Find the new values of μ and σ respectively required for each method to succeed, given that for the standardised normal distribution, given P(Z > 3.0902) = 0.0010.

*9. The end-of- month cashflow for a company can be modelled by the Normal distrribution with a mean of £10000 and a standard deviation of £7000. The treasurer is concerned about the possibility of a cash shortage and wishes to develop strategies to manage cash flow.
(i) What is the probability that monthly cash flow becomes negative?
(ii) The treasurer wants to establish a reserve fund that can be used in case of negative cash flow. Suppose they want to be able to cover all but 1% of the months. How much money should be in the reserve fund?
(iii) Determine the upper limit on cash flow that will be exceeded only 5% of the time.

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