UNIVERSITY OF BRISTOL : DEPARTMENT OF ECONOMICS

STATISTICS - MODULE 12122

EXERCISE 1 - RANDOM VARIABLES AND THEIR DISTRIBUTIONS

You should attempt as many questions as possible **before** your first tutorial (which starts in week 4). Although you should attempt all questions, however easy or difficult they appear, only starred questions will be discussed in tutorials, so it is important that you especially attempt these before your tutorials. Solutions to those not covered in tutorials will be provided later on in the term.

1. (i) Events A and B are such that $P(A) = \frac{5}{12}$, $P\left(A / \overline{B}\right) = \frac{7}{12}$, $P(A \cap B) = \frac{1}{8}$

Find (a) P(B) (b) P(A / B) (c) P(B / A)
(d) P($A \cup B$) (e) P($A \cap \overline{B}$) (f) P ($\overline{A} \cap \overline{B}$)

[Hint for (i) (f) : From Chapter 1, QM1, $n(A \cup B) + n(\overline{A} \cap \overline{B}) = n(S)$]

*(ii) Show that (a) $E\left(X - \mu\right)^2 = E(X^2) - [E(X)]^2 = E(X^2) - \mu^2$.

(b) Var ($a X + b$) = a^2 Var (X) .

2. Consider the following table of probabilities :

	Rain predicted (RP)	No rain predicted (NRP)	Total
Rain occurs (R)	0.18	0.02	0.20
No rain occurs (NR)	0.07	0.73	0.80
Total	0.25	0.75	1.00

(i) Determine the probability that it rains if rain is predicted.
(ii) Determine the probability that rain was predicted if it rains.
(iii) Express in words the probability whose value is 0.07 / 0.25.
(iv) Verify that P(R / RP) + P(NR / RP)=1.

3. An investment consultant on the basis of his past experience has suggested the following probability model for the rate of return of the investment.

Rate of return (%)	10	11	12	13	14	15	16
Probability	0.05	0.10	0.15	0.17	0.12	0.08	0.09

Rate of return (%)	17	18	19	20	21
Probability	0.06	0.05	0.05	0.04	0.04

(i) Define the random variable here.
(ii) Calculate the probability of a rate of return of less than 14%.
(iii) Calculate the probability of a rate of return of at least 14%
(iv) Calculate the expected return on this investment and the investment risk.

*4. The managing director of North Star Stoves is considering a project to increase production. The possible returns from the project are: £3000, £9000, £15000, and £21000 with probabilities 0.30, 0.20, 0.40 and 0.10 respectively.

(i) Define the investment probability model.

(ii) Find the probability that the return is (a) at least £15000 (b) between £9000 and £21000.

(iii) The firm's banker has asked for a single value for the return on investment. Select the appropriate summary measure and compute its value.

(iv) The managing director is interested in the variability of returns for the proposed investment . Calculate the standard deviation of returns.

(v) Constuct the cumulative distribution function $F(x)$ for this investment model and sketch it.

5. A continuous random variable X has a distribution with p.d.f. given by

$$f(x) = \frac{3}{16}(4 - x^2) \qquad 0 \le x \le 2$$
$$= 0 \qquad \text{otherwise}$$

(i) Sketch the p.d.f. curve.

(ii) Find the following probabilities (a) P ($X > 1$) (b) P ($0.5 \Leftrightarrow X \Leftrightarrow 1.7$)

(iii) Show that $E(X) = \frac{3}{4}$ and Var(X) = $\frac{19}{80}$.

6. A ceratin Economics professor has determined that the probability density function of disposable income is

$$f(x) = 0.2 \qquad 10 \le x \le 15$$

Another economist indicates that the savings function is $S = -2.0 + 0.2\,X$.

Find $E(X)$ and Var(X) and hence the mean and variance of savings, S.

[This is an example of the uniform distribution which will be discussed in Chapter 2, lecture notes)

*7. (i) What value must 'a' take if

$$f(x) = 0 \qquad x < 0$$
$$= ax \qquad 0 \le x \le \tfrac{1}{2}$$
$$= a(1 - x) \qquad \tfrac{1}{2} \le x \le 1$$
$$= 0 \qquad 1 < x$$

is to be a proper probability density function.

(ii) Sketch the p.d.f. function.

(iii) Calculate the following probabilities (a) P ($X > 0.25$), (b) P ($0.75 < X < 1.3$) .

(iv) Find $E(X)$ and Var (X).

(v) Find the cumulative distribution function, $F(x)$, corresponding to $f(x)$.

[Hint for part (iii) and $E(X)$: you can use calculus or geometrical arguments so look at sketch. Hint for part (v): the c.d.f. will have 4 parts to it as with $f(x)$] . **C. Osborne February 2000**