Assignment 1.

1. Suppose data is generated by the linear model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$ under the Gauss-Markov assumptions discussed in Chapter 6 in Greene and that we have obtained OLS estimates of the parameter vector $\boldsymbol{\beta}$.

a) What is the relationship between the observed OLS residuals $\mathbf{e}$ and the stochastic disturbance $\boldsymbol{\varepsilon}$. How do they relate to the independent variable matrix $\mathbf{X}$? Can $\mathbf{e}$ be used as an estimator of $\boldsymbol{\varepsilon}$?

b) Suppose that assumption 5 of non-stochastic $\mathbf{X}$ is relaxed. Explain the difference between a stochastic and a non-stochastic $\mathbf{X}$ matrix. How will it affect the properties of the OLS estimator?

2. Consider a model with one independent variable :

$$Y_i = \beta X_i + \varepsilon_i, i = 1, 2....., T \tag{1}$$

where the observations on X_i are non-stochastic and the disturbances are distributed independently and identically with mean zero and variance σ^2.

a) Derive the least squares estimator of β for this model without using matrix notation.

c) Show that the ratio of means estimator for β

$$b' = \frac{\overline{Y}}{\overline{X}} \tag{2}$$

where $\left(\overline{X}, \overline{Y}\right)$ are the arithmetic means of the observations (X_i, Y_i), satisfies the condition for linearity and unbiasedness in part (b). Show that it does not satisfy the conditions for minimum variance. Compare the variance of this estimator with the minimum that can be attained. When are the two variances the same.

d) Suppose that you drop m observations in calculating the least squares estimator. Show that it does not satisfy the conditions for minimum variance. Compare the variance of this estimator with the minimum that can be attained. When are the two variances the same.

e) Propose your own unbiased estimator of β (and name it after yourself!). Find also a biased estimator.

3. Consider a least squares regression of Y on k variables and a constant included in the $\mathbf{X}$ matrix of independent variables. Consider an alternative set of regressors, $\mathbf{Z} = \mathbf{XP}$, where $\mathbf{P}$ is a non-singular matrix. Thus, each column in $\mathbf{Z}$ is a mixture of some of the columns of $\mathbf{X}$. Prove that the residual vector in the regression of Y on $\mathbf{X}$ and Y on $\mathbf{Z}$ are identical. What relevance does this have to the question of changing the fit of a regression by changing the units of measurement of the independent variables?

4. a) Show that the R^2 for an OLS regression never decreases if you add more independent variables.

b) Explain why R^2 can be negative in the model of question 2.

5. In the least squares regression of Y on a constant and $\mathbf{X}$, in order to compute the regression coefficients on $\mathbf{X}$, we can first transform Y to the deviation with respect to the mean and, likewise, transform each column of $\mathbf{X}$ to the deviation from the respective column mean, then regress the transformed Y on the transformed $\mathbf{X}$ without constant. Do we get the same result if we only transform Y? What if we only transform $\mathbf{X}$?

6. Consider the human capital regression discussed on the first lecture

$$\ln w = \beta_1 + \beta_2 FEMALE_i + \beta_3 NSP_i + \mathbf{x}_i \gamma + \varepsilon$$

where $FEMALE = 1$ for females and 0 otherwise, NSP =1 for individuals with non-Swedish parents; $\mathbf{x}_i$ is a vector of three observable characteristics: number of years of education, years of work experience and years of work experience squared. The table below shows coefficient estimates and variable means.

	$\widehat{\beta}$	Means
Constant	3.77	
Female	-0.170	0.5097
Non-Swedish parents	-0.083	0.1106
Education	0.038	11.58
Experience	0.020	18.85
Experience^2	-0.0003	496.27
R^2	0.337	

Given model, calculate $E(\ln w \mid FEMALE = 1, NSP = 0, \mathbf{x} = \overline{\mathbf{x}})$