1. Define A as

$$\begin{pmatrix} 1 & 4 & 7 \\ 3 & 2 & 5 \\ 5 & 2 & 8 \end{pmatrix}$$

1.a. Determinant of A

$$(-1)^{1+1}\cdot 1\cdot(2\cdot 8 - 2\cdot 5) + (-1)^{2+1}\cdot 3\cdot(4\cdot 8 - 2\cdot 7) + (-1)^{3+1}\cdot 5\cdot(4\cdot 5 - 7\cdot 2) = -18$$

1.b. Trace of A

$$1 + 2 + 8 = 11$$

1.c. Inverse of A

$$\frac{1}{\left|\begin{pmatrix} 1 & 4 & 7 \\ 3 & 2 & 5 \\ 5 & 2 & 8 \end{pmatrix}\right|}\cdot\begin{bmatrix} (-1)^{1+1}\cdot(2\cdot 8 - 5\cdot 2) & (-1)^{1+2}\cdot(3\cdot 8 - 5\cdot 5) & (-1)^{1+3}\cdot(3\cdot 2 - 2\cdot 5) \\ (-1)^{2+1}\cdot(4\cdot 8 - 7\cdot 2) & (-1)^{2+2}\cdot(1\cdot 8 - 7\cdot 5) & (-1)^{2+3}\cdot(1\cdot 2 - 4\cdot 5) \\ (-1)^{3+1}\cdot(4\cdot 5 - 7\cdot 2) & (-1)^{3+2}\cdot(1\cdot 5 - 7\cdot 3) & (-1)^{3+3}\cdot(1\cdot 2 - 3\cdot 4) \end{bmatrix}^{T} = \begin{pmatrix} -0.333 & 1 & -0.333 \\ -0.056 & 1.5 & -0.889 \\ 0.222 & -1 & 0.556 \end{pmatrix}$$

2. Find the Cholesky Decomposition of a matrix given by

$$B := \begin{pmatrix} 25 & 7 \\ 7 & 13 \end{pmatrix}$$

First note

$$\begin{pmatrix} a & 0 \\ b & c \end{pmatrix}\cdot\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \rightarrow \begin{pmatrix} a^2 & a\cdot b \\ a\cdot b & b^2 + c^2 \end{pmatrix}$$

$$a := 5$$

$$b := \frac{7}{5}$$

$$c := \sqrt{13 - b^2}$$

$$c = 3.323$$

3. Jacobian for the system

$$y_1(x_1, x_2, x_3) := \ln\left(\frac{x_1}{x_2}\right)$$

$$y_2(x_1, x_2, x_3) := x_1 - x_2 + x_3$$

$$y_3(x_1, x_2, x_3) := x_1 \cdot x_2 \cdot x_3$$

$$J(x_1, x_2, x_3) := \begin{pmatrix} \frac{d}{dx_1} y_1(x_1, x_2, x_3) & \frac{d}{dx_2} y_1(x_1, x_2, x_3) & \frac{d}{dx_3} y_1(x_1, x_2, x_3) \\ \frac{d}{dx_1} y_2(x_1, x_2, x_3) & \frac{d}{dx_2} y_2(x_1, x_2, x_3) & \frac{d}{dx_3} y_2(x_1, x_2, x_3) \\ \frac{d}{dx_1} y_3(x_1, x_2, x_3) & \frac{d}{dx_2} y_3(x_1, x_2, x_3) & \frac{d}{dx_3} y_3(x_1, x_2, x_3) \end{pmatrix}$$

$$J(x_1, x_2, x_3) \rightarrow \begin{pmatrix} \frac{1}{x_1} & \frac{-1}{x_2} & 0 \\ 1 & -1 & 1 \\ x_2 \cdot x_3 & x_1 \cdot x_3 & x_1 \cdot x_2 \end{pmatrix}$$

4. For the matrix

$$X := \begin{pmatrix} 1 & 4 \\ 1 & -2 \\ 1 & 3 \\ 1 & -5 \end{pmatrix}$$

4.a. Compute $P = X(X'X)^{-1}X'$

$$X \cdot \left(X^T \cdot X\right)^{-1} \cdot X^T = \begin{pmatrix} 0.546 & 0.102 & 0.472 & -0.12 \\ 0.102 & 0.324 & 0.139 & 0.435 \\ 0.472 & 0.139 & 0.417 & -0.028 \\ -0.12 & 0.435 & -0.028 & 0.713 \end{pmatrix}$$

4.b. Compute M

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T = \begin{pmatrix} 0.454 & -0.102 & -0.472 & 0.12 \\ -0.102 & 0.676 & -0.139 & -0.435 \\ -0.472 & -0.139 & 0.583 & 0.028 \\ 0.12 & -0.435 & 0.028 & 0.287 \end{pmatrix}$$

4.c. Show P is idempotent

$$X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\cdot\left[X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\right] = \begin{pmatrix} 0.546 & 0.102 & 0.472 & -0.12 \\ 0.102 & 0.324 & 0.139 & 0.435 \\ 0.472 & 0.139 & 0.417 & -0.028 \\ -0.12 & 0.435 & -0.028 & 0.713 \end{pmatrix}$$

4.d. Show M is idempotent

$$\left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\right]\cdot\left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\right] = \begin{pmatrix} 0.454 & -0.102 & -0.472 & 0.12 \\ -0.102 & 0.676 & -0.139 & -0.435 \\ -0.472 & -0.139 & 0.583 & 0.028 \\ 0.12 & -0.435 & 0.028 & 0.287 \end{pmatrix}$$

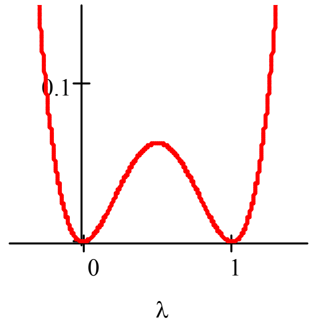
4.e. Show MP is zero

$$X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\cdot\left[\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} - X\cdot\left(X^T\cdot X\right)^{-1}\cdot X^T\right] = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

4.f. What are the roots of P

$$\left|\begin{pmatrix} 0.546-\lambda & 0.102 & 0.472 & -0.12 \\ 0.102 & 0.324-\lambda & 0.139 & 0.435 \\ 0.472 & 0.139 & 0.417-\lambda & -0.028 \\ -0.12 & 0.435 & -0.028 & 0.713-\lambda \end{pmatrix}\right| \to -.407000\text{e-}3\lambda + 1.000407\lambda^2 - 2.000\lambda^3 + \lambda^4$$

$- .407000e\text{-}3\cdot\lambda + 1.000407\cdot\lambda^2 - 2.000\cdot\lambda^3 + \lambda^4$



One of the roots must be zero. To find the rest we will successively divide by λ-the root just found and solve the resulting polynomial.

$$-.407000e\text{-}3\lambda + 1.000407\lambda^2 - 2.000\lambda^3 + \lambda^4$$

$$\lambda := .001$$

$$f(\lambda) := \frac{-.407000e\text{-}3\lambda + 1.000407\lambda^2 - 2.000\lambda^3 + \lambda^4}{\lambda}$$

$$r0 := \mathrm{root}(f(\lambda), \lambda)$$

$$r0 = 4.072 \times 10^{-4}$$

$$g(\lambda) := \frac{f(\lambda)}{\lambda - r0}$$

$$r1 := \mathrm{root}(g(\lambda), \lambda)$$

$$r1 = 1$$

$$h(\lambda) := \frac{g(\lambda)}{\lambda - 1}$$

$$r2 := \mathrm{root}(h(\lambda), \lambda)$$

$$r2 = 1$$

4.g. The roots of M. Since we know how to do it we'll let MathCad doi the drudgery.

$$\mathrm{eigenvals}\left(\begin{pmatrix} 0.454 & -0.102 & -0.472 & 0.12 \\ -0.102 & 0.676 & -0.139 & -0.435 \\ -0.472 & -0.139 & 0.583 & 0.028 \\ 0.12 & -0.435 & 0.028 & 0.287 \end{pmatrix}\right) = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 4.072 \times 10^{-4} \end{pmatrix}$$