Temple University
Department of Economics

Econometrics I
Distributions

1. Let Y be a discrete random variable with probability mass function

$$f(y) = P(Y = y) = \begin{cases} \dfrac{6}{(\theta y)^2}, & \text{if } y = 1,2,3,\ldots \\ 0, & \text{otherwise} \end{cases}$$

Show that the mean of Y does not exist.

2. Consider the random variables X and Y with means, variances and covariance denoted by $\mu_x,\ \mu_y,\ \sigma_x^2,\ \sigma_y^2$ and σ_{xy}, respectively. Suppose that Y is the return on a stock and X is the return on a bond such that $\mu_y > \mu_x$, $\sigma_y^2 > \sigma_x^2$. Finally, let W be a Bernoulli random variable distributed independently of X and Y, where P(W=1)=p, P(W=0)=1-p, and 0<p<1. Consider the random portfolio, or investment strategy, Z = WX + (1-W)Y.
   A. Find the mean of Z.
   B. Which is larger, the mean of Z or the mean of Y?
   C. Find the variance of Z.
   D. How large must $\mu_y - \mu_x$ be in order for $\sigma_z^2 > \sigma_y^2$?

3. Let the random variables Y_1 and Y_2 have a bivariate normal distribution with $E(Y_1) = 5$, $E(Y_2) = 10$, $Var(Y_1) = 1$, $Var(Y_2) = 25$, and $Cov(Y_1,Y_2) = 5\rho$. Find ρ such that $P(8<Y_2<12|Y_1=5) = .954$.