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An empirical note on the foundations of rational expectations

1. Introduction

In his 1961 article Muth proposes the rational expectations (RE) model on the grounds that the alternatives produce unstable and inconsistent forecasts of price changes. At that time the literature on the distribution of price changes, actual and anticipated, was scant. Judging from the surfeit of elegant rational expectations papers (Cukierman and Wachtel, 1982; Hercowitz, 1981; and Bordo, 1980 to name just three), one would think that the original Muth assessment of expectations and his subsequent formulation had gone unchallenged.

The evidence in refutation of the rational expectations paradigm is, in fact, both theoretical and empirical. Davidson (1982) has recently argued that forming expectations rationally will result in persistent errors of forecast. This conclusion stems from the RE assumption of the time independence of decisions and their consequences. The empirical evidence seems to support Davidson's evaluation of the RE hypothesis. Namely, most tests of price expectations either reject, or accept at considerable statistical expense, the RE null hypothesis of efficiency and consistency (Noble and Fields, 1982; Hafer and Resler, 1982; Pesando, 1975; Carlson and Parkin, 1975). Investigations of actual price changes show the distribution to be highly skewed and with fat tails, suggesting that many people are persistently fooled (Vining

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and Elwertowski, 1976; Buck and Gahlen, 1983). Finally, using models of aggregate economic activity, one can tell entirely different stories with essentially the same data. An example of this would be the recent papers by Mishkin (1982a; 1982b) and Barro (1977; 1978).[1] Thus, we are left with the continued use of an analogy of behavior which has little basis in the world it is meant to model. The purpose of the present paper is to provide further evidence, at the industry level, that the RE hypothesis does not fit the facts.

The present paper may be distinguished from previous empirical tests of the RE hypothesis in that it proposes a test of the "weakest" form of the RE hypothesis. The "strong" form of the hypothesis is that agents adjust their expectations to all relevant forms of publicly available information. The "weak" form of the hypothesis is based on adjustment to historical price series. Rejection of the latter implies rejection of the former. The usual procedure in the latter instance is to test for consistency and efficiency of expectations—essentially questions of central tendency and variance of the forecast. Implicit in the RE hypothesis, and its tests, is the notion that large numbers of people are not fooled and that no group is persistently fooled. That is, if price forecasts are made rationally in the Muthian sense, then actual price changes should not be skewed, nor should they deviate from a kurtosis of three. Thus, a "weakest" form test is the examination of the third and fourth moments of the price change distribution.

The order of the paper is as follows: Section 2 reviews the rational expectations hypothesis, its implications, and previous tests of the hypothesis. Section 3 proposes a new weakest form test. Conclusions are drawn in section 4.

2. A review of rational expectations models

The Rational Expectations Hypothesis (REH) states that deviations of employment and output from some equilibrium level are the result of errors in forecasting inflation. This result relies on the assumption that the subjective probability distribution of expected outcomes is the same as the objective probability distribution of actual outcomes. The disturbing implication of the RE hypothesis is that the central govern-

[1] Barro, in these papers, presents evidence for the neutrality of announced policy. Mishkin, arguing for "correct" lag length, comes to the opposite conclusion. Although the two authors do not use identical data sets, the level of aggregation is the same. The difference in conclusions is then a result of statistical technique and does little to clarify the RE controversy.

ment cannot influence real output or employment with systematic policy rules.

The typical model which leads to this rather disturbing conclusion posits a stochastic supply function with a trend component and a relative price component for each of *N* markets. The price in the *i*-th market is taken relative to the aggregate price level which agents in market *i* expect to prevail. These expectations are formed on the basis of information available to agents in the *i*-th market, which includes knowledge of the mean value of the overall (stochastic) price level but not its current realization. For purposes of tractability these models do not usually permit non-uniform supply elasticities.

Demand for the *i*-th commodity depends on the absolute price of the *i*-th good. In recent elegant papers (e.g., Cukierman and Wachtel, 1979; 1982) there are usually two Normal random disturbances added to the demand curve. One of these is unique to the *i*-th market and has no real economic interpretation; its essential purpose is to cause non-uniform expectations across markets. The other disturbance is common to all markets and is interpreted as the rate of change of nominal income. The mean of the second random variable can be controlled by the monetary or fiscal authority but is known to all economic agents.

During the current period agents in the *i*-th market observe the value of the disturbance unique to them, but cannot observe the realized current rate of growth of nominal income. Thus, although the overall price level can be computed for past periods as a weighted average of prices in the *N* markets, its current value is unknown.

The overall price level expected by agents in market *i* is formed as a weighted average of their price and the true (known) average of possible overall price realizations. The *i*-th price is found by equating supply and demand in that market. The weights are found by minimizing the mean square error (MSE) of the overall price forecast.

If the demand disturbances are normally distributed and agents use the MSE criterion to find the appropriate weights to use in their expectations model, then their forecast will be a best linear unbiased estimate. That is, they could use historical price data and least squares to find the weights. However, in the RE world with known structure of the economy, these weights can be computed from the disturbance variances.[2]

[2]The derivation, while straightforward, is somewhat tedious, but is available from the author. In any case, this property of being BLUE forms the basis for the consistency and efficiency tests of the weak form of the RE hypothesis.

Having computed the expected overall price level, we can show that the overall rate of inflation expected by agents in the i-th market depends on the average rate of growth of nominal income, the uniform elasticity of supply, and the disturbance variances, all of which are known.

The result of such models is that expectations about the general price level and rate of inflation are based on the systematic, or announced, component of central government policy; that is, they depend on the mean rate of growth of nominal income. Actions in individual markets are in turn based on the way in which announced policy impinges on the known structure of the economy. It is only the random component of demand and supply shocks which has real effects,[3] i.e., causes mistakes in relative prices and deviations from the natural rate of unemployment.

Direct tests of the RE hypothesis begin by taking expectations of the price level forecast across markets and substituting for the rate of change of nominal income, a function of prices, recursively to get the expected rate of inflation as an autoregressive process in past values of the price level. A test of the weak form of the RE hypothesis that expectations are based on the historical development of prices examines the coefficients of the statistical model:

$$\Pi_t = \sum_{i=1}^{n} a_i \Pi_{t-i} + U_t$$

(1)
$$\Pi_t^* = \sum_{i=1}^{n} a_i' \Pi_{t-i} + U_t$$

$$\Pi_t^* = a_1''\Pi_{t-1}^* + \sum_{i=2}^{n} a_i'' \Pi_{t-i} + U_t$$

for equality between equations, where Π is the actual rate of inflation, Π^* is the expected rate, and U_t is a disturbance term. The expected rate is usually computed as an inflation forecast averaged over survey respondents. Note that the current rate of inflation is a moving average of past values. The expected rate of inflation in the current period is also a moving average of past realized inflation rates. Finally, this period's forecast is a moving average of last period's forecast and previous realized inflation rates. The a_i, a_i', and a_i'' are model parameters to be estimated.

[3]On the real effects of anticipated and unanticipated inflation see Blejer and Leiderman (1980), Buck and Gahlen (1984), and Kawasaki, Gahlen, and Buck (1984).

It was noted above that under the usual assumption of rational expectations models the price level forecast would be optimal in the least squares sense. Hence, if $a_i = a_i'$ then Π_t^* will be a consistent estimate of Π_t. Similarly, if $a_i' = a_i''$ then Π_t^* will be an efficient estimate of Π_t.[4]

The consequence of consistency is that the expected rate of inflation converges in probability to the actual rate of inflation. Efficiency implies that there is no other forecasting model with smaller variance.[5] Symbolically, this is equivalent to

$$\text{(2)} \qquad \text{Cov}\,(U_t, \Omega_{t-1}) = 0$$

where U_t is an error term and Ω_{t-1} is lagged information. The information set includes lagged prices. If there are other regressors which should have been included in the model in (1) then (2) will not hold and some agent will be able to improve on least squares as a forecasting tool.

The implication of the efficiency and consistency criteria is that agents can learn about and incorporate new publicly available information. That is, not many of them are fooled and they do not make persistent mistakes on the basis of available information.

Thus the distribution of inflation forecasts will be normally distributed since a least squares forecast is a weighted average of sample means and by the central limit theorem the distribution of sample means is normal. This suggests that at the very least, when demand and supply shocks are assumed to be normal, the distribution of real price changes should be symmetric and they should have thin tails. An objective distribution of real outcomes with fat tails suggests that there are a lot of agents who have made incorrect forecasts; i.e., considerably more than 5 percent of the agents lie more than ± 1.96 standard deviations from the mean rate of inflation. In a world where RE reigns these people would quickly learn a better forecasting technique.

If the objective distribution is asymmetric then some agents have

[4]The forecast of the price *level* would be BLUE. The effect of first differencing to get the rate of inflation is to introduce serial correlation; hence we must settle for large sample properties.

[5]As an example, consider the sample median as an estimator of the population mean. The median is biased (a small sample property), but as sample size grows the median converges in probability on the population mean. However, as an estimator of the population mean, the sample mean has a smaller variance than the sample median.

incorrectly interpreted available information. That is, some agents have been either overly optimistic or overly pessimistic, thus pulling the mean forecast away from the mode. Again, a world ruled by RE would allow these agents to learn to correctly interpret what they had previously done wrong.

Furthermore, the possibility of persistent asymmetry poses great difficulty for the agents in the rational expectations model. Suppose that the structure of the economy is such that realized price changes are skewed. The three measures of central tendency are no longer equal, confronting the economic agent with a dilemma: should he stick to the rational expectations model and base his decisions on the least squares forecast of the mean of the overall rate of inflation or should he try to forecast one of the other two measures of central tendency? Making decisions on the basis of the forecasted mean could be risky as more than 50 percent of one's competitors will have a smaller price increase (in the case of positive skew). The problem is compounded if the direction of skew changes with the phases of the business cycle (see Vining and Elwertowski, 1976, on this possibility). Thus, apart from the tractability argument, there is an advantage of behavioral simplicity which speaks for symmetry of actual and expected price changes in the rational expectations world.[6]

In short, in the RE world there is no room for large numbers of large errors or for persistent bias.

Nevertheless, previous efforts to test the RE hypothesis have concentrated on the weak form of the proposition.[7] In his review of the literature of stock market efficiency, Fama (1970) asserts that expectations of stock prices and rates of return are formed rationally. This literature has relied on the objective distribution of securities prices and rates of return rather than the subjective distribution. Thus, out-

[6]To my knowledge there are no rational expectations models which assume asymmetric demand and/or supply shocks. If the shocks were asymmetric, agents continued to form expectations rationally, and agents were interested in the expected rate of inflation (in the mathematical sense), then actual price changes would have the same distribution as the demand shock variable and there would exist an RE equilibrium. But the asymmetry possibility raises the question of the appropriateness of the RE supply curve. For the *i*-th market the quantity supplied depends on the own price relative to the mathematical expected aggregate price level. But when the mode and median differ from the mean, is it still appropriate to compare one's price to that price expected to prevail in the aggregate?

[7]Rejection of the weak form of the hypothesis implies rejection of the strong form. See Pesando (1975) and Fama (1970).

comes are consistent with the RE model but there is not a test of the assumptions of the model.

The economic literature on expectations formation has had survey data sets available for the testing of the RE assumptions. The greatest effort has gone into the analysis of the Livingston data. On the basis of these data the results are mixed: Carlson (1977) and Pesando (1975) reject the RE hypothesis; Mullineaux (1978) fails to reject; Hafer and Resler (1982) divide the sample according to professional affiliation and fail to reject for all time periods. Using survey data from Michigan's Survey Research Center, Noble and Fields (1982) fail to reject the RE hypothesis. Carlson and Parkin (1975) and Foster and Gregory (1977) reject the RE hypothesis using qualitative response survey data for Great Britain. Finally, Kawasaki and Zimmerman (1983) reject the hypothesis for West Germany.

All of the empirical tests cited here are similar in that they are all based on the statistical model in (1) and consider only consistency and efficiency. Their common failing is that a forecasting rule may be consistent and efficient and still result in an objective distribution with large numbers of overly pessimistic and/or overly optimistic estimates. That is, rational expectations based rules are all consistent and efficient, but not all consistent and efficient rules need be based on rational expectations.[8]

3. A "weakest" test of the Rational Expectations Hypothesis

In the previous section it was argued that consistent and efficient forecasts are a necessary but not sufficient condition for the verification of the Rational Expectations Hypothesis. In addition, the logic and presumably intuitive appeal of the RE hypothesis lead one to expect that forecasting errors are neither large nor persistent in one direction or another. Thus, before pursuing elaborate tests of consistency and efficiency, one should examine the shape of the distribution of outcomes.

Three empirical distributions of price changes (outcomes) were

[8]Other studies of the shape of the distribution of outcomes include Vining and Elwertowski (1976) and Buck and Gahlen (1983). In both cases the distribution of outcomes was not normal. Bordo (1980) and Hercowitz (1981) are notable efforts to explain why prices may be sticky in an RE world, resulting in fat tailed, skewed distributions of outcomes.

Table 1

Forty industrial branches

Coal mining	Shipbuilding
Iron ore	Aerospace
Potash and rock salt	Electrical equipment
Mineral oil	Precision engineering
Other mining	Forging
Timber manufacture	Fine ceramics
Musical instruments	Glass
Stone and earth	Paper and board
Iron making	Printing
Foundries	Plastics
Drawing and cold rolling	Leather
Nonferrous metals	Leather goods
Chemicals	Shoes
Mineral oil refining	Textiles
Rubber and asbestos	Clothing
Saw mills	Grain milling
Wood chip	Edible oils
Structural steel	Sugar
Machinery	Brewing
Motor vehicles	Other food

constructed from annual West German data (1950–1977) for forty industries (see Table 1) in order to provide a test of the RE hypothesis. The three distributions constitute nominal price changes, real price changes, and price changes about the rate of inflation. The rate of change of the *i*-th price is given by

$$DP_{it} = \ln P_{it} - \ln P_{it-1}$$

and the rate of inflation is given by

$$DP_t = \sum_{i=1}^{40} W_{it}\, DP_{it}$$

where the W_{it} are two-year moving averages of the *i*-th industry's share of total sales. The phase moments of the three distributions are calculated from the formulas in Table 2.

Table 2

	Nominal	Real	About the rate of inflation
Mean (μ_i)	$\frac{1}{27} \Sigma DP_{it}$	$\frac{1}{27} \Sigma (DP_{it} - DP_t)$	$\frac{1}{27} \Sigma (DP_{it} - DP_t)$
Variance (σ_i^2)	$\frac{1}{27} \Sigma (DP_{it} - \mu_i)^2$	$\frac{1}{27} \Sigma ((DP_{it} - DP_t) - \mu_i)^2$	$\frac{1}{27} \Sigma (DP_{it} - DP_t)^2$
m-th Moments	$\frac{1}{27\sigma^m} \Sigma (DP_{it} - \mu_i)^m$	$\frac{1}{27\sigma^m} \Sigma ((DP_{it} - DP_t) - \mu_i)^m$	$\frac{1}{27\sigma^m} \Sigma ((DP_{it} - DP_t)^m$

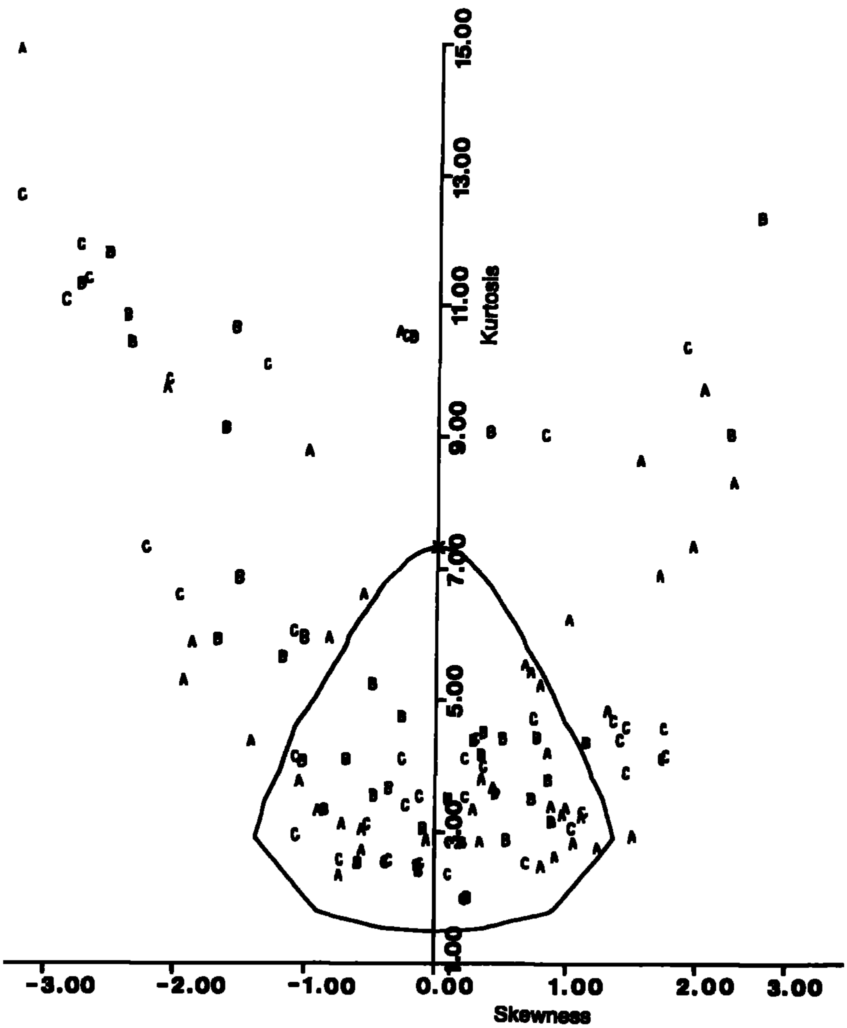

Figure 1

Moments of the price change distribution

By construction, the mean of the means of nominal rates of price change will equal the aggregate rate of inflation; the mean of the means of real price changes and changes about the rate of inflation will be zero. In and of itself the variance tells us little. Thus, the phase means and variances are not reported. The third (skewness) and fourth (kurtosis) moments of the distributions are plotted in Figure 1. The nominal, real, and inflation centered distributions are plotted by A, B, and C, respectively.

For comparison, the Normal distribution has zero skewness coefficient and kurtosis equal to three. As is readily seen, most West German industries have price change distributions which are peaked and with fat tails (kurtosis greater than three). This suggests that over a 25-year period many industries have missed the actual overall rate of price change with considerable frequency. The large skewness coefficients indicate that some industries are persistently fooled in the implementation of their expectations. Also plotted in Figure 1 is a 99 percent confidence contour (Bowman and Shenton, 1975) for the test of normally distributed price changes.

The 99 percent confidence contour has an interpretation similar to that of a confidence interval. If the skew-kurtosis pair computed from a sample of size n falls within the confidence contour then we are 99 percent confident that the sample was drawn from a normally distributed population. If the sample skew-kurtosis pair do not fall within the contour then it is very unlikely that the sample was drawn from a normal population. Note that as the confidence level decreases from, say, 99 percent to 90 percent, the area covered by the confidence contour in the diagram would become larger.

The null hypothesis of normality is rejected between 15 and 19 times, out of a possible 40, for each of the 3 formulations of the price change distribution. This is a very unlikely result under the normality assumptions of the RE model. Further, the scatter of points suggests that many industries have skewed price change distributions with fat tails. When the mode, median, and mean price change all differ from one another it doesn't seem likely that economic agents are making decisions using the rules assumed in the RE model.

These results are particularly problematic for the RE model given the high rate of real growth, low rate of inflation, and stable government policy in Germany for the period under consideration.

4. Conclusions

The empirical evidence presented here further undermines the RE hypothesis. That the empirical distribution of industrial price changes has fat tails suggests that large numbers of firms are persistently frustrated in their interpretation of the available information and that they do not share a common economic model. Also, the highly skewed distributions suggest that the system of relative prices has changed significantly over the last twenty-five years in West Germany. The drift in relative prices seriously handicaps the plausibility of an ergodic world necessary for the RE hypothesis.

REFERENCES

Barro, R. "Unanticipated Money Growth and Unemployment in the U.S." *American Economic Review*, 1977, *67* (1), 101–115.

———. "Unanticipated Money, Output, and the Price Level in the U.S." *Journal of Political Economy*, 1978, *86* (3), 549–580.

Blejer, M., and Leiderman, L. "On the Real Effects of Inflation and Relative Price Variability: Some Empirical Evidence." *Review of Economics and Statistics*, 1980, *62*, 539–544.

Bordo, M. D. "The Effects of Monetary Change on Relative Commodity Prices and the Role of Long Term Contracts." *Journal of Political Economy*, 1980, *88* (6), 1088–1109.

Bowman, K. O., and Shenton, L. R. "Omnibus Test Contours for Departures From Normality Based on $\sqrt{b_1}$ and b_2." *Biometrika*, 1975, *62* (2), 243–250..

Buck, A., and Gahlen, B. "Relative Preise, Inflation und gesamtwirtschaftliche Aktivitaet." In *Makrooekonomik heute-Gemeinsamkeiten und Gegensatze*. Ed. by G. Bombach and B. Gahlen, Schriftenreiche des Wirtschaftswissenschaftlichen Seminars Ottobeuren, Tuebingen, 1983.

———. "The Real Effects of Price Variability in the Federal Republic of Germany: 1953-1977." *Economics Letters*, 1984, *15*, 79–85.

Carlson, J. A. "A Study of Price Forecasts." *Annals of Economic and Social Measurement*, 1977, *6* (1), 27–56.

Carlson, J., and Parkin, M. "Inflation Expectations." *Economica*, 1975, *42*, 123–135.

Cukierman, A., and Wachtel, P. "Differential Inflationary Expectations and the Variability of the Rate of Inflation." *American Economic Review*, September 1979, *69* (4), 595–609.

———. "Relative Price Variability and Nonuniform Inflationary Expectations." *Journal of Political Economy*, 1982, *90* (1), 146–155.

Davidson, P. "Rational Expectations: A Fallacious Foundation for Studying Crucial Decision-Making Processes." *Journal of Post Keynesian Economics*, Winter 1982-83, *5* (2), 182–198.

Fama, E. "Efficient Capital Markets: A Review." *Journal of Finance*, May 1970, 383–417.

Foster, J., and Gregory, M. "Inflation Expectations: The Use of Qualitative Survey Data." *Applied Economics*, 1977, *9* (2), 319–329.

Hafer, R. W., and Resler, D. H. "On the Rationality of Inflation Forecasts: A New Look at the Livingston Data." *Southern Economic Journal*, 1982, *48* (4), 1049–1056.

Hercowitz, Z. "Money and the Dispersion of Relative Prices." *Journal of Political Economy*, 1981, *89* (2), 328–356.

Kawasaki, S.; Gahlen, B.; and Buck, A. "The Variability of Relative Price Expectations, the Rate of Inflation and the Phillips Curve." In *Leading Indicators and Business Cycle Surveys*. Ed. by K. H. Oppenlander and G. Poser. England: Gower Publishing Co., 1984.

Kawasaki, S., and Zimmerman, K. "Testing the Rationality of Price Expectations for Manufacturing Firms," Berlin: IIM, mimeo, 1983.

Mishkin, F. "Does Anticipated Aggregate Demand Policy Matter? Further Econometric Results." *American Economic Review*, 1982a, *72* (4), 788–802.

———. "Does Anticipated Monetary Policy Matter? An Econometric Investigation." *Journal of Political Economy*, 1982b, *90* (1), 22–51.

Mullineaux, D. J. "On Testing for Rationality: Another Look at the Livingston Price Expectations Data." *Journal of Political Economy*, 1978, *86* (2), 329–336.

———. "Inflation Expectations and Money Growth in the United States." *American Economic Review*, 1980, *70* (1), 149–161.

Noble, N. R., and Fields, T. W. "Testing the Rationality of Inflation Expectations Derived from Survey Data: A Structure Based Approach." *Southern Economic Journal*, 1982, *49* (2), 361–373.

Pesando, J. E. "A Note on the Rationality of the Livingston Price Expectations Data." *Journal of Political Economy*, 1975, *83* (3), 849–858.

Vining, D. R., and Elwertowski, T. C. "The Relationship between Relative Prices and the General Price Level." *American Economic Review*, 1976, *66* (4), 699–708.