ANDREW J. BUCK
Temple University
Philadelphia, Pennsylvania

*Inflation and Price Change Variability: Some New Evidence from Old Data**

The research presented here uses annual data for Germany (1850–1914) and the United States (1881–1913) to explore the relationship between inflation, relative price variability and output. No previous study has used data from this period. The purpose of the paper is to consider the (neo)classical proposition that the price level and relative prices are independent. The impact of price uncertainty on output is also considered. The study shows that there is a strong relationship between inflation and price variability, though it is not consistent across countries. There seems to be little role for relative price change variance in explaining changes in output.

1. Introduction

In recent years there has been an exceptional interest in the variability of relative price changes, the rate of inflation, and the effect on the real economy. This literature was spawned by the now classic Vining and Elwertowski (1976) study of the systematic relationship between the rate of inflation and the variance of relative price changes.[1] At the close of their paper they raise the issue of a causal link between inflation, price variability, and the real economy.

The research presented here uses annual data for Germany (1850–1914) and the United States (1881–1913) to explore the relationship between inflation, relative price variability, and output. No previous study has used data from this period. Graham (1930), Hercowitz (1981), and Mills (1927) have used interwar data. Studies using more modern German and U.S. data include Fischer (1981, 1982), Froyen and Waud (1987), and Buck and Gahlen (1983).

The purpose of the paper is to consider, with this new though chronologically old data, the (neo)classical proposition that the price

*Thanks go to Bernhard Gahlen, Klaus Gerhaeusser, anonymous referees and seminar participants at Temple University for comments on an earlier draft. The research reported here was supported in part by the Deutsche Forschungsgemeinschaft. The views presented here are solely those of the author.

[1]This literature is reviewed in Marquez and Vining (1984) and Gerhaeusser (1987).

Journal of Macroeconomics, Summer 1990, Vol. 12, No. 3, pp. 415–426
Copyright © 1990 by Louisiana State University Press
0164-0704/90/$1.50

level and relative prices are independent.[2] Subsequently, the impact of price variability on output is also considered.[3] Consistent with other studies, it is found that there is a strong relation between inflation and relative price change variance. However, there seems to be little role for relative price change variance in explaining changes in output.[4] The results are especially interesting since the data are for a period predating conscious, activist fiscal and monetarist policy in either country.[5]

2. The Evidence

The U.S. data has been assembled from Commons (1902), Kemmerer (1909), the U.S. Department of Commerce (1975), and the U.S. Bureau of Labor (1899–1905). The data base consists of annual observations on money supply, output and price indices for 65 commodities. The rate of inflation and relative price change variance were computed as

$$\pi_t = \frac{1}{65} \sum_i (\ln P_{it} - \ln P_{it-1}) , \tag{1}$$

and

$$PVAR_t = \frac{1}{64} \sum_i [(\ln P_{it} - \ln P_{it-1}) - \pi_t]^2 , \tag{2}$$

[2]In fact, this proposition is quite relevant to the new classical macroeconomics. Bordo (1980), Cukierman (1984), Cukierman and Wachtel (1982), Hercowitz (1981), Parks (1978), and Taylor (1981) are representative of efforts to bend the new models to the U.S. postwar experience.

[3]In some instances, notably in Friedman's Nobel lecture, variability and uncertainty are used interchangeably. Following Cukierman and Wachtel (1982), we note that there is a distinction.

[4]Only in Hesselman (1983) has the relationship between relative price change variance and inflation also failed to emerge. This same paper finds no evidence of association between relative price change variance and either unemployment or output.

[5]At the point in time covered by this data set there was not a central bank in either country. Further, as understood by a Keynesian, there was no role for central government spending in economic stabilization. The conscious use of monetary and fiscal policy to achieve particular macroeconomic goals probably did not begin until the interwar years.

where P_{it} is the price index for the ith commodity at time t. Were they available, the terms in the sums would be weighted by expenditure shares.[6]

The German data are compiled from Hoffman (1965). There are 43 price indices and an output measure for the entire period.[7] The money supply is available only for the period 1865–1913. The inflation rate and relative price change variance are computed as in (1) and (2).

For comparable periods, the German inflation rate shows less variation, in terms of either the range or variance of the series, as does the series for the variance of relative price changes. For the entire period the rate of inflation was −0.67% per year for the U.S. and 0.9% for Germany. The rate of growth of output averaged 3.4% for Germany and 4.6% per year for the U.S. Finally, the growth of the money supply in the U.S., at 5.6% per annum, was more than twice that of Germany.

Table 1 contains regression results for the regression of relative price change variance on current and lagged inflation. To state the hypothesis succinctly, the relation between the general price level and relative price variability should be positive and may be attributed to costs of price adjustment, market disequilibrium, or nominal contracts (see Parks 1978; Cukierman 1982, 1983; Cukierman and Wachtel 1982; or Fischer 1981). The lag structure is not determined on the basis of theory, but is driven by the data, and hence the estimated models are reduced form specifications. But note that the reported equations do not differ in specification from Parks' (1978) structural model. All regressions are corrected for autocorrelation using a maximum likelihood grid search routine. By any measure, the results for the U.S. are more significant and suggest that higher rates of inflation result in greater relative price variability.[8] Although much weaker, the opposite result is found for

[6]See Gerhaeusser (1986) for a discussion of proper weighting and its empirical importance.

[7]The output measure is net social product at factor cost. It is roughly equivalent to national income computed from labor income, rent, interest, and profits.

[8]Using the test statistics suggested by Sims (1980), the model was tested for temporal precedence. For both countries, inflation precedes price variance, albeit weakly. Using the test statistics in Granger and Newbold (1986) it was found that a three-period lag was not a restriction over four periods in a two-variable vector autoregressive model of inflation and price change variance. A result for contemporary Germany is in Gerhauesser (1988). More elaborate VAR and cointegration techniques would have overwhelmed the scant number of available observations.

TABLE 1. *Relative Price Change Variance and Inflation*

		Constant	Trend	π_t	π_{t-1}	π_{t-2}	π_{t-3}	*Rho*	R^2
USA	1	0.0399	−0.0003	0.0247				0.6236	
		(2.38)	(−0.67)	(0.62)				(4.52)	0.42
	2	0.0420	−0.0005	0.1094	0.1442			0.4376	
		(3.98)	(−1.44)	(2.61)	(3.52)			(2.60)	0.57
	3	0.0435	−0.0005	0.1153	0.1554	0.0244		0.4118	
		(4.11)	(−1.57)	(2.68)	(3.54)	(0.64)		(2.34)	0.58
	4	0.0438	−0.0005	0.1159	0.1565	0.0258	0.0046	0.4067	
		(3.91)	(−1.52)	(2.64)	(3.48)	(0.64)	(0.11)	(2.26)	0.58
Germany	1	0.0113	−0.0000	−0.0074				0.2893	
		(4.20)	(−0.51)	(−0.31)				(2.34)	0.09
	2	0.0114	−0.0000	0.0022	−0.0473			0.3119	
		(4.06)	(−0.43)	(0.09)	(−1.95)			(2.50)	0.15
	3	0.0108	−0.0000	0.0009	−0.0506	0.0122		0.3028	
		(3.72)	(−0.25)	(0.04)	(−1.98)	(0.49)		(2.37)	0.15
	4	0.0105	−0.0000	0.0055	−0.0434	0.0047	0.0326	0.2847	
		(3.55)	(−0.23)	(0.21)	(−1.64)	(0.18)	(1.28)	(2.18)	0.17

NOTES: The variable π denotes the rate of change of the price level. *Rho* is the $AR(1)$ error parameter estimate. t-statistics are in parentheses. The reported R-square is corrected for the loss of degrees of freedom.

Germany. This seeming conflict is also found in Fischer's (1982) study of postwar data. Part of the explanation lies in the evolution of the two inflation series. The inflation rate in Germany is far more frenetic than that of the U.S.[9] In fact, regressions of price change variance on current and lagged values of the absolute value of inflation lead to results the opposite of those in Table 1 (see Fischer 1981 for the justification). The absolute value of inflation has a minor role in explaining price change variance in the U.S., while it has a significant role in Germany.[10] In any case, one may conclude that more inflation or deflation adds noise to the system of relative price changes. This result is consistent with Friedman's (1977) assertion in his Nobel address, but is somewhat at variance with classical economics. The results in Table 1 are also somewhat troublesome for the new macroeconomics, which subscribes to neutrality of money paradigms. While the U.S. results are consistent with studies using more contemporary U.S. data, upon which Cukierman and others have predicated their models, the inflation coefficient sign reversal for Germany is troublesome. Models in the spirit of Cukierman would require, depending on country and period, different signs on the error coefficients that drive their money or demand shock specification.[11] This, of course, weakens the general applicability of their theoretical models.

Table 2 presents results from regressing output growth (the Frickey Index for the U.S. and Net Social Product for Germany) on the variance of relative price change and inflation.[12] The moti-

[9]A time series may evolve frenetically, though it may have a small variance. A series that is positively serially correlated will evolve smoothly. A series that is negatively serially correlated will evolve frenetically, that is, exhibit frequent sign reversals, although it may have the same variance as its smoothly evolving counterpart.

[10]The result for Germany is

$$\begin{array}{lllll} P\hat{V}AR_t = & 0.0039 + & 0.00003t + & 0.1392|\pi_t| & R^2 = 0.30\,. \\ & (1.26) \quad & (0.54) \quad & (4.34) & Rho = 0.38 \end{array}$$

Higher order lags on inflation were never significant. Lagged values of *PVAR* were never significant.

[11]For example, in Cukierman (1983) the money supply and productivity are serially independent in their first differences. Therefore, money and productivity are negatively serially correlated. Excess demand shocks are assumed to be uncorrelated. The result is a relationship between inflation and price variability that is robust to a number of model parameter changes, but not to the changes in the structure of shocks.

[12]The inflation–price change variability and price variability–output hypotheses

vating hypothesis, suggested by Friedman (1977) and developed by Cukierman (1984), is that the increasing price change variance introduces added noise to the information transmitted by relative prices and consequently reduces output growth. The empirical specification can be interpreted as a Phillips Curve with the addition of price change variance as a measure to capture price uncertainty (Levi and Makin 1980). This hypothesis is tested for contemporary U.S. and German data by Blejer and Leiderman (1980) and Buck and Gahlen (1983), respectively. Briefly, both studies find that increased price variance reduces output growth and that it is unanticipated, not anticipated, inflation which increases output growth. Fitting their specifications to the present 19th-century data produces quite different results.[13] In the U.S., most output growth is explained by contemporaneous inflation.[14] Although an F-statistic for the set of price variance coefficients is significant for Equations 2, 3, and 4 of Table 2 (F = 3.65, 3.38, 2.71, respectively), the t-statistics for the individual coefficients are not significant, nor are linear combinations of the coefficients significant (t = −0.01, 0.24, 0.22, respectively). This result is quite different from previous studies.[15]

In Germany, output growth is explained by current and past inflation. Only first and second order lags on the price change variance variable are significant. Consistent with work using contemporary data for the U.S. and Germany, these coefficients are significantly less than zero.

Table 3 is similar to Table 2, except that the inflation variable

---

(Note cont. from p. 419)

have been treated as distinct, though related, issues in the literature. This paper continues that tradition.

Failing to find a relation in one instance does not preclude finding a relation in the other. In the best of all possible worlds, such things would not happen because the data would be properly prepared and internally consistent. But that certainly is not the case of the nineteenth century.

[13]The lag structures reported in Blejer and Leiderman (1980) and Buck and Gahlen (1983) and the present work all differ slightly. First, the theory does not suggest a particular lag structure or length. Second, the Buck and Gahlen paper uses quarterly data. In any event, Blejer and Leiderman report results for contemporary, one- and two-period lags on price variability. Their results differ from those reported here in that they restricted, without statistical tests, the lag coefficients to be equal.

[14]"Explained" is used rather loosely here. The conclusion presumes that one had previously accepted the monetarist proposition as being true.

[15]In regressions not reported here, the number of business failures and dollars involved were used as dependent variables as measures of "real" economic activity.

TABLE 2. *Price Change Variance and Output*

R^2		Constant	Trend	Output $(t-1)$	$PVAR(t)$	$PVAR$ $(t-1)$	$PVAR$ $(t-2)$	$PVART$ $(t-3)$	$\pi(t)$	$\pi(t-1)$	$\pi(t-2)$	$\pi(t-3)$	Rho
USA	1	0.1319	−0.0018	−0.3585	−0.5633				1.2133				−0.1647
		(2.85)	(−1.47)	(−1.40)	(−0.64)				(4.08)				(−0.54)
0.44													
	2	0.0919	−0.0012	−0.2370	−0.0105	0.1159			1.1818	−0.4393			−0.2919
		(1.22)	(−0.81)	(−0.53)	(−0.01)	(0.09)			(3.51)	(−0.72)			(−0.62)
0.44													
	3	0.0626	−0.0006	−0.3024	0.3574	−0.4748	0.7168		1.1412	−0.4692	−0.1733		−0.2805
		(0.63)	(−0.33)	(−0.28)	(0.24)	(−0.27)	(0.62)		(3.18)	(−0.35)	(−0.21)		(−0.26)
0.46													
	4	0.0668	−0.0007	−0.2861	0.3629	−0.5256	0.8551	−0.1965	1.1393	−0.4848	−0.1612	0.0324	−0.3047
		(0.49)	(−0.29)	(−0.11)	(0.22)	(−0.25)	(0.37)	(−0.09)	(2.53)	(−0.16)	(−0.07)	(0.07)	(−0.12)
0.46													
Germany	1	0.0210	0.0001	0.2527	−0.3987				0.6169				−0.5952
		(1.99)	(0.38)	(2.17)	(−0.69)				(5.61)				(−4.65)
0.44													
	2	0.0439	0.0001	−0.5831	0.8838	1.3398			0.5285	0.6945			0.1898
		(2.35)	(0.27)	(−3.83)	(1.11)	(1.75)			(3.56)	(3.61)			(1.01)
0.48													
	3	0.0539	0.0002	−0.6991	0.6141	−1.0766	−1.2572		0.4651	0.7637	0.1332		0.3562
		(2.21)	(0.46)	(−5.92)	(0.77)	(−1.39)	(−1.61)		(3.07)	(4.29)	(0.87)		(2.32)
0.50													
	4	0.0636	−0.0000	−0.6603	0.8116	−0.8604	−1.5567	0.0544	0.4360	0.7398	0.1998	−0.3461	0.3072
		(2.55)	(−0.00)	(−5.51)	(1.06)	(−1.14)	(−2.00)	(0.07)	(2.83)	(4.28)	(1.33)	(−2.34)	(1.87)
0.56													

NOTES: See notes to Table 1. The variable *PVAR* denotes the variability of relative prices, per Equation (2) in the text.

TABLE 3. *Unanticipated Inflation, Price Change Variance and Output*

		Constant	Trend	Output $(t-1)$	$PVAR(t)$	$PVAR$ $(t-1)$	$E\pi_t$	$E\pi_{t-1}$	$U\pi_t$	$U\pi_{t-1}$	Rho	R^2
USA	1	0.2147	−0.0038	−0.5288	−0.8572	−0.3069	2.4569		0.2699		−0.1476	0.66
		(4.53)	(2.96)	(−2.85)	(−0.79)	(−0.34)	(6.15)		(0.75)		(−0.61)	
	2	0.1628	−0.0030	−0.3569	−0.6003	0.1343	2.4016	−0.6156	0.3330	−0.0971	−0.3255	0.68
		(2.21)	(−1.93)	(−1.08)	(−0.54)	(0.12)	(5.86)	(−0.92)	(0.92)	(−0.25)	(−0.89)	
	3	0.0724	−0.0007	−0.1034			1.6069	−0.9771	0.4064	−0.1518	−0.2777	0.62
		(2.11)	(−0.79)	(−0.42)			(5.57)	(−2.56)	(1.27)	(−0.48)	(−0.97)	
Germany	1	0.0913	−0.0007	−0.8211	0.2219	−0.7371	0.4353		0.0129		0.1545	0.57
		(3.72)	(−1.08)	(−3.06)	(0.68)	(−1.98)	(3.22)		(0.88)		(0.96)	
	2	0.0637	−0.0005	−0.7301	0.1966	−0.6771	0.4052	0.1806	0.2357	0.1965	0.1932	0.62
		(2.44)	(−0.96)	(−2.91)	(0.54)	(−1.88)	(1.66)	(0.73)	(2.32)	(1.98)	(1.01)	
	3	0.0458	−0.0002	−0.5610			1.9694	0.2477	0.3269	0.3735	0.2656	0.57
		(1.98)	(−0.41)	(−3.11)			(4.44)	(0.52)	(2.04)	(2.08)	(1.25)	

NOTES: t-statistics are in parentheses. See notes to Tables 1 and 2.

has been decomposed into anticipated ($E\pi_t$) and unanticipated ($U\pi_t$) components.[16] The method used was to construct expected inflation from the fitted values of a regression with the history of money and prices on the right hand side. The unanticipated inflation series is then the residuals from this regression.[17] For the U.S., it is only expected inflation which significantly affects output growth, either in the short or long run. For Germany, both anticipated and unanticipated inflation have a positive impact on output growth.

(Note cont. from p. 420)

In both cases, inflation reduced business failures (and their size), while increased price change variance increased business failures. Similar data is not available for Germany.

[16]Lagged expectations in this specification can be justified on several grounds. First, they capture the present consequences of real decisions taken in the past on the basis of their current expectations. Overlapping contracts and time-to-build models would be consistent with this interpretation. Second, the available information set at time t consists of the history of prices. Decomposition of price into expected and unexpected components, as done in the paper, is orthogonal, and so in no way changes the information set.

[17]This technique is quite commonplace and is used in Blejer and Leiderman (1980) and Buck and Gahlen (1983). The result for the U.S. is

$$\underset{}{\hat{\pi}_t} = \underset{(-4.95)}{-0.1038} + \underset{(2.59)}{0.0014t} - \underset{(-1.82)}{0.4175\pi_{t-1}} + \underset{(3.68)}{0.4677M_t} + \underset{(3.10)}{0.5912M_{t-1}} .$$

The result for Germany is

$$\underset{}{\hat{\pi}_t} = \underset{(0.10)}{0.0018} - \underset{(-0.23)}{0.0001t} + \underset{(0.12)}{0.0527\pi_{t-1}} + \underset{(1.61)}{0.1777M_t} + \underset{(1.45)}{0.2165M_{t-1}} ,$$

where π is the inflation rate, M is the growth of the money supply, and t-statistics are in parentheses. Lag length was chosen on the basis of an F-test. A t-statistic would have been inappropriate because of the high degree of collinearity between the lagged inflation and money series.

While there was not a central bank in either country, as we understand it, the concept of the money supply is still meaningful. The data used here is most closely related to today's notion of M2. In both countries a bank had to subscribe to central government bonds as part of its capitalization. In the U.S. during the period in question the banks could issue their own notes but, by and large, did not do so. The National Bank Act of 1863 levied a tax on banks that did not have a national charter. As a consequence, most banks held such a charter and were allowed to issue a national currency depending on their capitalization. The Reichsbank had a monopoly on note issue in Germany, though joint stock banks could issue close substitutes. For precise definitions of the money supply see Hoffman (1965) and the U.S. Department of Commerce (1975).

3. Conclusions

A result of the early models of the "new" classical macroeconomists was that changes in the price level should be independent of relative price change variance. Following the work of Vining and Elwertowski, a wave of theoretical papers has shown that the (apparent) causation should be from inflation to relative prices and that the association should be positive. Using nineteenth-century data, this relationship is shown to hold for the U.S. However, the association is negative for the same period for Germany.

Related to the issue of inflation and relative price change is the effect of this price system noise on real output (Friedman 1977). The Blejer and Leiderman (1980) result that increased inflation of the unanticipated variety increases output, while increased price change variance reduces it, does not hold for the U.S. prior to establishment of the Federal Reserve Bank. The results for Germany of the same period are as expected, except that anticipated inflation also matters. The statistical cause for this dissimilarity is in the evolution of the different price series. The root cause, however, probably lies in the differing political and economic developments in Germany and the U.S.[18] In any case, the neoclassical hypotheses are not empirically robust.

Received: March 1988
Final version: August 1989

References

Blejer, Mario, and Leonardo Leiderman. "On the Real Effects of Inflation and Relative Price Variability: Some Empirical Evidence." *Review of Economics and Statistics* 62 (May 1980): 539–44.

Bordo, Michael. "The Effects of Monetary Change on Relative Commodity Prices and the Role of Long Term Contracts." *Journal of Political Economy* 88 (December 1980): 1088–1109.

Buck, Andrew, and Bernhard Gahlen. "On the Real Effects of Relative Price Changes in the Federal Republic of Germany: 1950–1977." *Economic Letters* 11 (June 1983): 231–36.

[18]Germany was not unified until 1871 under Bismarck, while the U.S. was already 100 years old at that point. The geographic expansion of Germany was not possible in the same way as the U.S. The development of railroads was quite different in the two countries (Hoffman 1965; Stolper 1966).

Commons, John R., and N.I. Stone. *Reports of the U.S. Industrial Commission.* Vol. 19, 1101–13. Washington: GPO, 1902. Reprint. Westport, Connecticut: Greenwood Press, 1970.

Cukierman, Alex, and Paul Wachtel. "Relative Price Variability and Nonuniform Inflationary Expectations." *Journal of Political Economy* 90 (February 1982): 146–57.

Cukierman, Alex. "Relative Price Variability, Inflation and the Allocative Efficiency of the Price System." *Journal of Monetary Economics* 9 (March 1982): 131–62.

———. "Relative Price Variability and Inflation: A Survey and Further Results." *Carnegie-Rochester Series on Public Policy* 19 (1983): 103–58.

———. *Inflation, Stagflation, Relative Prices and Imperfect Information.* New York: Cambridge University Press, 1984.

Fischer, Stanley. "Relative Shocks, Relative Price Variability and Inflation." *Brookings Papers on Economic Activity* 2 (1981): 381–441.

———. "Relative Price Variability and Inflation in the U.S. and Germany." *European Economic Review* 18 (March 1982): 171–96.

Friedman, Milton. "Nobel Lecture: Inflation and Unemployment." *Journal of Political Economy* 85 (June 1977): 451–72.

Froyen, Richard T., and Roger N. Waud. "An Examination of Aggregate Price Uncertainty in Four Countries and Some Implications for Real Output." *International Economic Review* 28 (June 1987): 353–72.

Gerhaeusser, Klaus. "Ideal Divisia Moments, Price Dispersion and Inflation." *Economic Letters* 20 (June 1986): 227–32.

———. "Inflation and Relative Price Variability" (in German). Ph.D. thesis, University of Augsburg, West Germany, 1987.

———. "Price Dispersion and Inflation: A Test of Causality." *Applied Economics* 20 (October 1988): 701–09.

Graham, Frank D. *Exchange, Prices, and Production in Hyperinflation: Germany, 1920–23,* Princeton, N.J.: Princeton University Press, 1930.

Granger, Clive W.J., and Paul Newbold. *Forecasting Economic Time Series.* 2d ed. New York: Academic Press, 1986.

Hercowitz, Zvi. "Money and the Dispersion of Relative Prices." *Journal of Political Economy* 89 (April 1981): 328–56.

Hesselman, Linda. "The Macroeconomic Role of Relative Price Variability in the U.S.A. and U.K." *Applied Economics* 15 (April 1983): 225–33.

Hoffman, Walter G. *Das Wachstum der Deutschen Wirtschaft Seit der Mitte des 19. Jahrhunderts.* New York: Springer-Verlag, 1965.

Kemmerer, Edwin. *Money and Credit Instruments in Their Relation to General Prices.* 2d. ed. New York: Henry Holt and Co., 1909.

Levi, Michael D., and John H. Makin. "Inflation Uncertainty and the Phillips Curve: Some Empirical Evidence." *American Economic Review* 70 (September 1980): 1022–27.

Marquez, Jaime, and Daniel Vining. "Inflation and Relative Price Behavior: A Survey of the Literature." *Economic Perspectives: An Annual Survey of Economics* 3 (1984): 1–56.

Mills, Frederick C. *The Behavior of Prices.* New York: NBER, 1927.

Parks, Richard. "Inflation and Relative Price Variability." *Journal of Political Economy* 86 (February 1978): 79–96.

Sims, Christopher. "Macroeconomics and Reality." *Econometrica* 48 (January 1980): 1–48.

Stolper, Gustav. *The German Economy: 1870 to the Present.* New York: Harcourt, Brace and World, 1966.

Taylor, John. "On the Relation Between the Variability of Inflation and the Average Inflation Rate." *Carnegie-Rochester Conference Series on Public Policy* 15 (Autumn 1981): 57–85.

U.S. Department of Commerce. *Historical Statistics of the United States: Colonial Times to 1970.* Bureau of the Census, Washington, D.C., 1975.

U.S. Bureau of Labor. *Bulletin.* Nos. 38, 51, 54, 57, 59, 75, and 77. Washington, D.C.: GPO, 1899–1905.

Vining, Daniel, and Thomas Elwertowski. "The Relationship Between Relative Prices and the General Price Level." *American Economic Review* 66 (September 1976): 699–78.